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Simplicial analysis of the water resource distribution system of an enrichment plant

Yevhen Tymokhin*

Postgraduate Student
Kryvyi Rih National University
50027, 11 Vitalii Matusevich Str., Kryvyi Rih, Ukraine
https://orcid.org/0009-0003-0821-584X

Vadym Kharlamenko

PhD in Technical Sciences, Associate Professor
Kryvyi Rih National University
50027, 11 Vitalii Matusevich Str., Kryvyi Rih, Ukraine
https://orcid.org/0000-0002-4008-5518

Abstract. Water management in modern ore enrichment plants requires the application of advanced topological analysis methods to optimise their complex information and control architecture prior to implementing intelligent control systems based on machine learning algorithms. Consequently, the main aim of the work was to comprehensively investigate structural connectivity and determine the exact control hierarchy of a water resource distribution system within an enrichment plant section using the methodology of simplicial analysis to further minimise specific energy consumption. The research was based on the fundamental principles of algebraic topology. The primary object was a topological graph model of the water supply system, which was systematically decomposed into thirteen key concepts, including technological mechanisms, sensor arrays, and control modules at both local and upper levels. The study yielded significant results, confirming the high effectiveness of the proposed mathematical approach. For the developed system, an adjacency matrix was constructed, and the structural redundancy index was calculated, resulting in a positive value of $R_k \approx 0.31$. This strictly mathematically confirmed the presence of high connectivity, the absence of structural breaks, and robust internal reservation of the technological network. Furthermore, a step-by-step multidimensional analysis was performed: the topological dimension of each individual simplex was determined, the corresponding equivalence classes were formed, and the first structural vector of the topological complex $Q_k = (13,3,1)$ was generated. The calculation of the uniform distribution index of directed graph links ($\epsilon^2 = 4.77$) proved the existence of a rigid hierarchical structure with pronounced centres of influence, completely refuting the hypothesis of network homogeneity. It was established that the adaptive optimisation module and integrated actuators formed the highest topological dimensions ($q = 1$ and $q = 2$). The practical value of the obtained results lies in forming a reliable topological basis for the future deployment of graph neural networks. Identifying the simplices of the highest dimension allows for optimising the hardware placement of sensors, significantly increasing the reliability of local control loops, and reducing the specific consumption of electrical energy and fresh water through targeted management of hydraulic flows under conditions of dynamically changing physical characteristics of the input ore

Keywords: water supply; algebraic topology; graph theory; adjacency matrix; automation; recycled water; mineral processing

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**Corresponding author (eugene.t@knu.edu.ua)*

Introduction

Modern ore enrichment plants operate as complex, multi-level technological networks where water acts as both the primary transport and technological medium. Effective water management, maintaining stable closed-cycle recirculation, and minimising specific energy consumption are fundamental to the operational resilience and economic efficiency of the mining industry. The inherent presence of massive circulating loads and dynamic fluctuations in input ore characteristics demand rapid, automated balancing of hydraulic flows. Consequently, the transition from traditional, rigid control schemes to adaptive, intelligent management architectures has become a strategic priority for ensuring sustainable and resource-efficient mineral processing. The relevance of this study is driven by the urgent industrial need to improve energy efficiency at enrichment plants by identifying hidden topological patterns in closed-loop water networks prior to machine learning integration.

During the 2024-2025 period, the optimisation of industrial networks has been extensively driven by artificial intelligence. For instance, G. Vittori *et al.* (2025) studied graph neural networks for modeling water distribution networks and established their superior capability in processing non-Euclidean topological data. To address the challenge of limited monitoring, J. Wang *et al.* (2025) focused on pressure estimation and concluded that transfer learning with graph neural networks drastically improves prediction accuracy. Furthermore, Z. Li *et al.* (2025) developed a pipeline leakage detection method based on graph convolutional networks, highlighting its high localisation accuracy in complex distribution networks. Regarding wastewater management, G. Baskar *et al.* (2024) analysed various strategies and proved that machine learning techniques effectively optimise treatment processes. This was also supported by N.D. Suzaimi *et al.* (2025), who demonstrated that machine learning significantly enhances industrial wastewater treatment efficiency. Contemporary research on leakage detection in water distribution systems focuses on the application of machine learning methods and spatial data integration to improve the accuracy and responsiveness of monitoring processes. In particular, Gaurav & S. Rathi (2025) investigated a machine learning-based approach for identifying leaks, aiming to automate anomaly detection in water distribution data. The authors demonstrated that machine learning models can improve leak detection performance compared to traditional approaches, especially in conditions characterised by noisy and complex datasets. A similar direction is explored by D. Elshazly *et al.* (2024), who proposed an automated GIS-based spatial machine learning method for leak detection in water distribution networks using sensor data. The study highlights that integrating geographic information systems with machine learning techniques enhances the ability to account for the spatial structure of the network and improves the accuracy of leak localisation.

Despite the significant progress in deploying intelligent algorithms, the direct application of graph neural networks in the highly specific environment of ore enrichment plants remains challenging. Effective implementation requires a rigorous mathematical formalisation of the hidden hierarchical connections within the physical network. Without a clear topological abstraction, predictive models struggle to weigh the relative influence of different operational nodes. Simplicial analysis serves as a fundamental mathematical tool to bridge this gap, allowing for the assessment of structural redundancy and the identification of controlling hub concepts within multi-connected infrastructure.

Thus, the main aim of this work was to comprehensively study the structural connectivity, evaluate structural redundancy, and determine the control hierarchy of the water resource distribution system of an enrichment plant section using simplicial analysis methods. To achieve this aim, the following specific tasks were formulated: to decompose the technological scheme of the enrichment section into formal concepts and construct an adjacency matrix reflecting material and information flows; to perform a multidimensional q-analysis and calculate the structural redundancy indicators; to identify the simplices of the highest mathematical dimension to definitively determine the control hierarchy of the system.

Materials and Methods

The primary object of the study was the water resource distribution system of a typical, fully operational section of an ore enrichment plant. Based on the system analysis of the enrichment technology, the complex technological scheme was systematically decomposed into a finite set of interconnected concepts (graph nodes). The topological model of the system was presented in the form of a directed graph, the vertices of which represented both the technological apparatuses and hydraulic nodes directly, as well as the elements of the automatic control system governing them.

The physical process under investigation included the continuous supply of water at multiple sequential stages. The first stage involved primary grinding using ball mills (type MShTs-3600×5500) working in a closed cycle with single-spiral mechanical classifiers (type KS-24×125). Following this, the material passed through wet magnetic separation using drum separators (type PBM-120/300). Subsequent stages of secondary and tertiary grinding also utilised MShTs-3600×5500 mills, but classification was performed using cylindrical-conical hydrocyclones (type GTSK-710). Desliming, a crucial step for removing fine barren rock, was conducted in magnetic deslimers (type MD-9A), and final moisture removal was executed by disk vacuum filters (Smyrnov

& Biletskyi, 2002). The used water, now in the form of a heavily saturated pulp, entered technological sumps and was subsequently pumped via heavy-duty slurry pumps into the massive tailing dump for natural settling and clarification (Airapetian, 2010).

To evaluate the operational efficiency, a rigorous system of balance equations for the water-slurry scheme was applied. The fundamental amount of water in any given product W_n (measured in m^3/h) was determined as:

$$W_n = Q_n \cdot R_n, \tag{1}$$

where Q_n – the mass of the dry solid product (t/h); R_n – the pulp dilution ratio. In the baseline operational scenario, the initial ore feed for the primary grinding stage was set at 98 t/h. However, due to the circulating load of sands from the spiral classifiers, the actual feed into the primary mills could reach up to 131 t/h. This required rigid, highly responsive hierarchical automation to dynamically adjust the water supply in real-time (Matsui *et al.*, 2013).

To conduct the simplicial analysis and construct the mathematical model, a formalised designation of the set of system concepts $K = \{K_1, K_2, \ldots, K_{13}\}$ was systematically introduced. These 13 concepts comprehensively covered the information and control architecture of the plant and were divided into three functional groups. The first group represented physical and environmental nodes, which included: K_1 – Water sources (fresh technical water); K_2 – Water preparation (first-lift pumping stations); K_3 – Reservoirs (buffer tanks for clean and recycled water); K_4 – Distribution network (main pipelines and manifolds); K_5 – Water consumers (grinding, classification, and separation units); K_6 – Wastewater collection (sumps and slurry pumps); K_7 – Tailing dump (natural clarification process); K_8 – Recycled water pumping station. The second group consisted of informational and control nodes, specifically: K_9 – Sensors (flow meters, level gauges, radiometric density meters); K_{10} – PLC/SCADA (local programmable logic controllers); K_{11} – Adaptive-optimisation module (high-level predictive server); K_{12} – Actuators (variable-frequency drives on pump motors, control valves). Finally, the third group accounted for external factors, specifically uncontrollable disturbances like changes in input ore characteristics, hardness, and iron content (K_{13}). The visual representation of the established topological links between the system's conceptual nodes is shown in Figure 1.

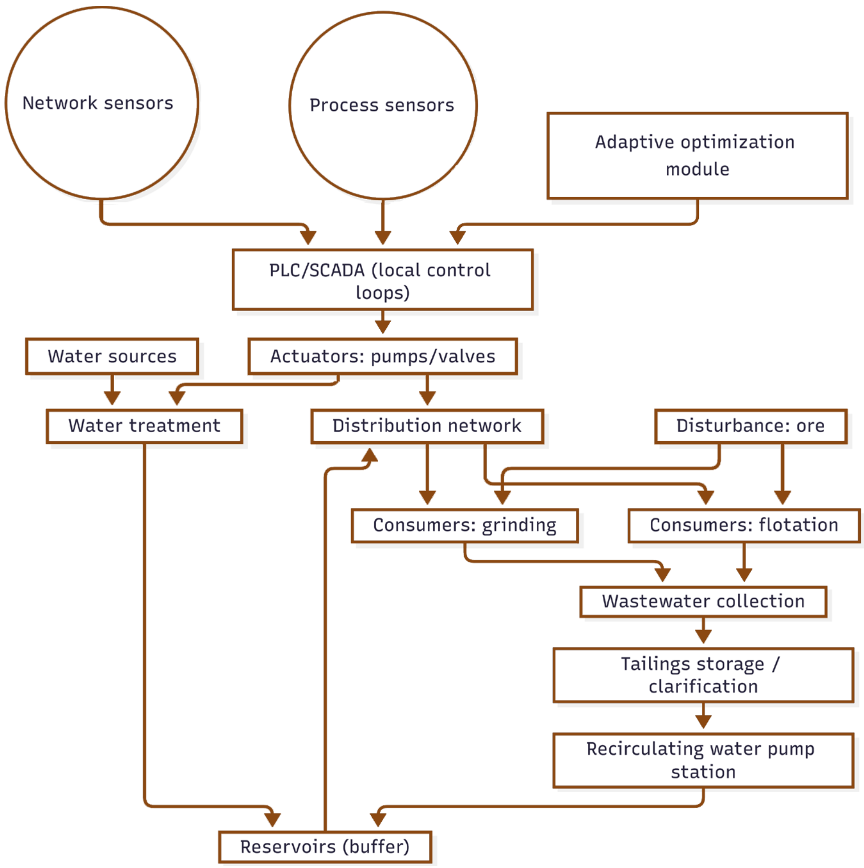

Figure 1. Topological graph model of the water resource distribution system

Notes: the graph visualises the 13 identified concepts and the directed informational and hydraulic flows between them, serving as the basis for the adjacency matrix

Source: developed by the authors

The structural connectivity of this defined set was evaluated using the q-connectivity algorithm (Salieva & Yaremchuk, 2020). The methodology required the construction of an adjacency matrix $\Lambda = [\lambda_{ij}]$ of dimension 13×13. The rule for forming the matrix was strictly deterministic: $\lambda_{ij} = 1$ if the i-th node was a direct source of material or informational influence on the j-th node (a directed edge from i to j), and $\lambda_{ij} = 0$ otherwise.

In this formally defined scenario, the adjacency matrix took the precise form:

$$\Lambda = \begin{bmatrix} 0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1&0\\ 0&1&0&1&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0 \end{bmatrix}. \quad (2)$$

Based on the adjacency matrix, the structural redundancy indicator R_k was calculated using the formula:

$$R_k = \frac{1}{m_2}\sum_{i=1}^{m_2}\sum_{j=1}^{m_2}\lambda_{ij} - 1, \quad (3)$$

where m_2 – the total number of graph vertices ($m_2 = 13$). A positive value for this indicator ($R_k > 0$) served as a mathematical criterion confirming that the system was connected without structural breaks and possessed reliable internal redundancy.

For the step-by-step q-analysis, the rows of the adjacency matrix were interpreted as simplices. The topological dimension of the simplex q_i associated with the i-th vertex was determined strictly by its out-degree minus one:

$$q_i = \rho_{out}(i) - 1. \quad (4)$$

To properly interpret the resulting hierarchy, specific functional criteria were established for the dimensional levels. The base level $q = 0$ consisted of nodes acting strictly as physical executors, passive storage, or single-direction data sources without broader routing capabilities. The intermediate level $q = 1$ included nodes responsible for multi-channel information collection or local control logic. Finally, the highest-dimension nodes $q \geq 2$ were classified as executive hubs, which possessed multiple outgoing control links and dictated the dynamic behaviour of the entire underlying system.

Finally, to definitively classify the network topology, the index of uniform distribution of directed graph links (ϵ^2) was calculated using the variance of the out-degrees:

$$\epsilon^2 = \sum_{i=1}^{m_2}(\rho_{out}(i) - \rho_{cp})^2, \quad (5)$$

where ρ_{cp} – the arithmetic average degree of the graph vertices. The criterion for evaluating this index was defined as follows: a value approaching exactly zero ($\epsilon^2 \approx 0$) indicates a perfectly homogeneous system (e.g., a simple ring network without distinct control centres). Conversely, a value significantly greater than zero (typically $\epsilon^2 > 3.0$) strictly classifies the structure as highly heterogeneous, mathematically proving the existence of a rigid, star-like control hierarchy with pronounced centers of influence that require prioritised optimisation.

Results and Discussion

Applying the water-slurry balance equations formulated in the methodology, the operational limits of the primary grinding stage were quantitatively determined prior to topological modelling. For the baseline continuous ore feed of $Q = 98$ t/h and an optimal pulp dilution ratio of $R_n = 3.0$ (required for effective magnetic separation), the fundamental technological water demand was calculated as $W_n = 98 \cdot 3.0 = 294$ m^3/h. However, during periods of peak circulating loads from the spiral classifiers, the actual solid feed into the primary mills dynamically surged to $Q = 131$ t/h. To maintain the exact same dilution ratio of $R_n = 3.0$, the water supply had to be instantaneously adjusted to $W_n = 131 \cdot 3.0 = 393$ m^3/h. This massive physical fluctuation of nearly 100 m^3/h within a single control loop strongly indicates that static hydraulic balancing is highly inefficient, directly necessitating a highly adaptive, hierarchically structured information and control topology.

Based on the formalised adjacency matrix constructed according to the proposed methodology, the derived topological data were extracted for further quantitative evaluation. The total number of directed edges in the graph (m_1) representing the material and informational links was exactly 17. With a total of 13 vertices ($m_2 = 13$), the structural redundancy indicator was calculated as $R_k = \frac{17}{13} - 1 \approx 0.31$. The positive value of this indicator strictly proved the high connectivity of the system and the definite presence of structural redundancy without breaks. In the context of industrial automated control systems, such redundancy is a distinct sign of the potential reliability of the system and its robust ability to reconfigure routing under conditions of severe technological disturbances.

Table 1. Dimensions of simplices for the nodes of the water supply system graph

Concept	Description	Out-degree (ρ_{out})	Simplex dimension (q_i)
K_1	Water sources (fresh technical water from rivers/wells)	1	0
K_2	Water preparation (first-lift pumping stations)	1	0

Table 1. Continued

Concept	Description	Out-degree (ρ_{out})	Simplex dimension (q_i)
K_3	Reservoirs (massive buffer tanks for clean and recycled water)	1	0
K_4	Distribution network (main pipelines, manifolds, valves)	1	0
K_5	Water consumers (grinding, classification, and separation units)	1	0
K_6	Wastewater collection (sumps, heavy-duty slurry pumps)	1	0
K_7	Tailing dump (water settling and natural clarification process)	1	0
K_8	Recycled water pumping station (returning water to the plant)	1	0
K_9	Sensors (flow meters, level gauges, radiometric density meters)	2	1
K_{10}	PLC/SCADA (local programmable logic controllers for direct loop control)	1	0
K_{11}	Adaptive-optimisation module (high-level balance control server)	2	1
K_{12}	Actuators (frequency converters on pumps, control valves)	3	2
K_{13}	Disturbances (changing input ore characteristics, hardness, iron content)	1	0

Notes: the table presents the calculated out-degrees and simplex dimensions for each conceptual node within the proposed topological model
Source: developed by the authors

Following the dimensional analysis of the out-degrees for each concept, the structural vector of the complex Q_k was formed. This vector effectively reflected the number of mathematically connected components participating in the formation of the hierarchy at each discrete dimensional level q. The resulting vector was $Q_k = (13,3,1)$. This rigorously reflected the management architecture of the plant: at the highest executive level of physical influence ($q = 2$), there was exactly one complex node consisting of the integrated actuators (K_{12}); at the intermediate informational level ($q = 1$), there were three distinct nodes dedicated to information collection and global optimisation (K_9, K_{10}, K_{11}); and at the base foundational level ($q = 0$), all 13 technological and environmental nodes participated as the groundwork of the complex.

To definitively confirm the non-homogeneous nature of the structure, the index of uniform distribution of links of the directed graph (ϵ^2) was calculated using the variance of the out-degrees. Given that the average vertex degree was approximately 1.31, the computation yielded $\epsilon^2 \approx 4.77$. Since the calculated value significantly exceeded the standard threshold of 3.0, it mathematically proved that the system was fundamentally not homogeneous. It possessed pronounced, high-degree hub nodes that forced a rigid, star-like management hierarchy upon the underlying physical hydraulic system.

For a more detailed topological analysis and identification of structural bottlenecks in the network, the constructed adjacency matrix λ was further examined from the perspective of vertex in-degree. The column-wise analysis of the matrix indicates that the highest topological load is associated with the wastewater collection node K_6 and the technological consumer block K_5. In the context of dynamic processes within the beneficiation plant, these components act as pronounced topological bottlenecks. They aggregate the largest share of incoming material, hydraulic, and information flows, which makes them particularly vulnerable to cumulative process disturbances. From a hydraulic and process engineering perspective, this concentration of inflows implies a reduced degree of operational redundancy in these nodes. In particular, any perturbation propagating through upstream subsystems is amplified at these convergence points due to the superposition of multiple flow streams. This effect is further intensified under non-stationary operating conditions, where variability in flow rates and processing delays leads to temporal desynchronisation of interconnected units. Specifically, node K_6 receives discharged hydraulic streams from multiple processing stages. Any asynchrony in equipment operation during observed peak flow fluctuations with amplitudes of approximately 100 m^3 per hour may trigger instantaneous overflow of technological sumps and cause an emergency shutdown of the closed-loop system. In such conditions, even short-term deviations from nominal operating regimes can initiate cascading imbalances across the downstream sections of the network. Node K_6 demonstrates a similar vulnerability pattern, as it serves as a major aggregation point for technological consumers with heterogeneous demand profiles. The interaction between variable input streams and fluctuating consumption intensifies internal pressure variability and increases the likelihood of transient overload states. Thus, the mathematical identification of recipient nodes with maximal ρ *in* demonstrates that the deployment of high-dimensional control simplices, actuators of level $q = 2$ and sensors of level $q = 1$, should be spatially and logically concentrated around these bottleneck areas. This configuration ensures targeted damping of incoming flows and reliably prevents cascade failures within the hydraulic network.

The identification of actuators (K_{12}) as the highest-dimension simplex ($q = 2$) fundamentally guides the theoretical optimisation process for the plant. Since water supply pumps currently often rely on deterministic relay-contact logic (simple on/off switching), which results in excessive pipeline pressure, uncoordinated water delivery, and sump overflows during the 100 m/h fluctuations, the physical modernisation must be strictly targeted at these specific nodes. The obtained mathematical model allows to theoretically recommend upgrading traditional direct-on-line pump starters to variable-frequency drives (VFDs) and electrically controlled modulating valves. Theoretically, these advanced $q = 2$ actuators would dynamically adjust the pump speed based on continuous real-time signals from the $q = 1$ sensor arrays. This targeted hardware intervention is projected to prevent destructive pump surging and stabilise the exact water-to-solid ratio required by the grinding mills, which would result in substantial theoretical savings in specific fresh water consumption.

Historically, water supply at enrichment plants was designed using strictly deterministic, static balance models. For instance, T.S. Airapetian (2010) investigated the mechanical and hydraulic layouts of these systems, concluding that the primary design focus should be on the static volumetric capacities of sumps to buffer incoming flows. Similarly, V.O. Smyrnov & V.S. Biletskyi (2002) evaluated the maximal throughput of heavy-duty slurry pumps, establishing it as the fundamental criterion for continuous operation. A.K. Zapolskyi (2005) analysed industrial water quality and emphasised the absolute necessity of massive circulating water volumes to maintain baseline technological parameters without considering dynamic adjustments. Furthermore, early control strategies were largely limited to basic relay-contact logic, as discussed by A.M. Matsui *et al.* (2013), who demonstrated that simple on/off switching was the standard for drum mills. Finally, L.F.R. Camargo *et al.* (2018) highlighted the persistent trade-off between productivity and quality in iron ore mining simulations under static control, proving that without adaptive systems, output quality inevitably fluctuates. Current findings fundamentally differ from these historical models. By applying simplicial analysis, it was mathematically proved that the network possesses a hidden multidimensional hierarchy. While traditional methods struggled to handle the extreme 100 m^3/h hydraulic fluctuations caused by circulating loads, current topological identification of executive hubs provides a theoretical basis for dynamic, real-time balancing.

The targeted theoretical optimisation of actuators identified by current structural vector directly supports contemporary energy efficiency and resilience goals. M. Bras *et al.* (2025) investigated the cost-efficiency of continuous pump scheduling in water supply systems, concluding that optimisation models significantly reduce energy costs compared to manual operation. Current identification of $q = 2$ hubs provides the exact topological targets where such advanced scheduling technologies should be deployed. Furthermore, X. Tian *et al.* (2017) applied multi-scenario predictive control for water resources, demonstrating that ensemble forecasting improves operational reaction times to disturbances. Their conclusions confirm current hypothesis that managing external disturbances (K_{13}) requires focusing predictive algorithms on key control nodes. Regarding network stability, Y. Yang & E.Y. Lo (2025) explored the resilience of interdependent infrastructure systems using synthetic mathematical modelling, proving that topological interdependencies are crucial for maintaining continuous service during disruptions. Similarly, A. Ponti *et al.* (2021) proposed graph-based vulnerability metrics specifically for urban water distribution networks, establishing that evaluating structural redundancy is vital for identifying critical failure points. Current calculated positive structural redundancy indicator ($R_k \approx 0.31$) serves as a micro-level equivalent to their macro-level resilience metrics, mathematically confirming the enrichment plant's inherent ability to withstand severe technological disturbances.

In contrast to rigid historical routing, this topological abstraction perfectly prepares the system for modern artificial intelligence, which aligns with recent advanced studies. X. Fan *et al.* (2022) developed a deep reinforcement learning model for resilient water distribution networks, concluding that artificial intelligence can drastically improve repair and rerouting decisions. X. Zhao *et al.* (2024) formulated chance-constrained optimal power-water flow assisted by deep neural networks, demonstrating that iterative algorithms successfully manage the energy-water nexus uncertainties. G. Vittori *et al.* (2025) highlighted the exceptional capabilities of graph neural networks in processing spatial data for complex networks, confirming that they outperform traditional models in predicting hydraulic states. However, predictive models require a physically meaningful topological foundation to function accurately, as noted by J. Wang *et al.* (2025), who concluded that transfer learning relies heavily on structural similarities between networks. Current methodology provides exactly this missing deterministic foundation by mapping the exact informational weights of each node prior to training. Furthermore, Z. Li *et al.* (2025) developed a pipeline leakage detection method based on graph convolutional networks, emphasising that the algorithm's accuracy strictly depends on the strategic placement of monitoring equipment. S. Yang *et al.* (2025) explored anomaly management using artificial intelligence, proving that reliable data collection from specific nodes is required to prevent false alarms. Finally, N.D. Suzaimi *et al.* (2025) demonstrated that intelligent control and machine learning drastically improve overall industrial wastewater treatment efficiency. Current topological analysis complements all these studies by providing a mathematically rigorous criterion for sensor and actuator deployment, theoretically dictating that smart equipment must be concentrated exclusively around the highest-dimension simplices ($q \geqslant 1$) to maximise the effectiveness of future system integration.

Conclusions

The comprehensive topological analysis of the water resource distribution system at the enrichment plant section successfully demonstrated the profound necessity and effectiveness of mathematical formalisation in modern industrial engineering. By applying the principles of algebraic topology, the complex technological network was systematically decomposed and evaluated, moving decisively beyond traditional, flat hydraulic models. The quantitative assessment of structural redundancy, yielding a positive indicator of $R_k \approx 0.31$, mathematically verified that the network is highly connected, robust, and inherently capable of dynamic rerouting without structural breaks. This redundancy is critical for maintaining theoretical operational stability during periods of severe technological disturbances, such as the massive hydraulic fluctuations reaching nearly 100 m/h caused by circulating sand loads.

Furthermore, the multidimensional q-analysis and the calculation of the uniform distribution index ($\epsilon^2 \approx 4.77$) definitively refuted the assumption of network homogeneity. Instead, it mathematically proved the existence of a rigid, star-like control hierarchy. By explicitly identifying the system's actuators as the highest-dimension executive hubs ($q = 2$) and the sensor arrays as the intermediate informational nodes ($q = 1$), the study provided a precise theoretical blueprint for physical modernisation. This theoretical identification recommends the targeted implementation of variable-frequency drives and real-time adaptive control. If implemented according to this topological model, destructive pump surging could be theoretically eliminated, and the exact water-to-solid ratio required by the grinding mills would be stabilised, which is projected to lead to a substantial reduction in specific electrical energy and fresh water consumption. The theoretical results obtained in this study establish a rigorous, deterministic foundation for the future digitalisation of the mineral processing industry. The primary prospect for future research lies in integrating these mathematically derived topological abstractions directly into the training pipelines of artificial intelligence systems, specifically graph neural networks. By providing machine learning algorithms with a physically meaningful structural map prior to deployment, future research can focus on developing fully autonomous, predictive control architectures that are highly resistant to data anomalies and operational blind spots.

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Conflict of Interest

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Симпліціальний аналіз системи розподілу водних ресурсів збагачувальної фабрики

Євген Тимохін

Аспірант
Криворізький національний університет
50027, вул. Віталія Матусевича, 11, м. Кривий Ріг, Україна
https://orcid.org/0009-0003-0821-584X

Вадим Харламенко

Кандидат технічних наук, доцент
Криворізький національний університет
50027, вул. Віталія Матусевича, 11, м. Кривий Ріг, Україна
https://orcid.org/0000-0002-4008-5518

Анотація. Управління водним господарством сучасних рудних збагачувальних фабрик вимагає застосування новітніх методів топологічного аналізу для оптимізації їхньої складної інформаційно-керуючої архітектури перед впровадженням інтелектуальних систем керування на базі алгоритмів машинного навчання. З огляду на це, головною метою роботи було комплексне дослідження структурної зв'язності та визначення точної ієрархії управління системи розподілу водних ресурсів секції збагачувальної фабрики із застосуванням методології симпліціального аналізу для подальшої мінімізації питомих енерговитрат. Дослідження базувалося на фундаментальних принципах алгебраїчної топології. Головним об'єктом виступала топологічна граф-модель системи водопостачання, яка була декомпозирована на тринадцять ключових концептів, що включали технологічні апарати, датчики та керуючі модулі локальних і верхніх рівнів. Отримані результати підтвердили високу ефективність запропонованого математичного підходу. Для розробленої системи було побудовано матрицю суміжності та розраховано показник структурної збитковості, який склав додатне значення $R_k \approx 0{,}31$. Це строго математично підтвердило наявність високої зв'язності, відсутність розривів та надійне внутрішнє резервування технологічної мережі. Крім того, виконано покроковий багатовимірний аналіз: визначено розмірність кожного окремого симплексу, сформовано відповідні класи еквівалентності та згенеровано перший структурний вектор топологічного комплексу $Q_k = (13,3,1)$. Розрахунок індексу рівномірного розподілу зв'язків орієнтованого графа $\epsilon^2 = 4{,}77$ довів наявність жорсткої ієрархічної структури з яскраво вираженими центрами впливу, повністю спростувавши гіпотезу про однорідність мережі. Встановлено, що найвищу топологічну розмірність формували саме адаптивно-оптимізаційний модуль та інтегровані виконавчі механізми ($q = 1$ та $q = 2$). Практична цінність одержаних результатів полягає у тому, що вони сформували надійне топологічне підґрунтя для майбутнього розгортання графових нейронних мереж. Визначення симплексів найвищої розмірності дозволяє оптимізувати апаратне розміщення датчиків, суттєво підвищити надійність локальних контурів регулювання та знизити питомі витрати електричної енергії і свіжої води за рахунок таргетного управління гідравлічними потоками в умовах динамічної зміни фізичних характеристик вхідної руди

Ключові слова: водопостачання; алгебраїчна топологія; теорія графів; матриця суміжності; автоматизація; оборотна вода; переробка корисних копалин